An Echo Canceller with Reduced Arithmetic Precision

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Abstract—It is well known that implementation of adaptive digital filters for echo cancellation in full-duplex transmission over telephone lines requires a large number of bits for the tap representation. This is due to the large dynamic ranges of the echo and far-end signal. In this paper, we introduce a new echo canceller: the "controlled gain echo canceller" (CGEC) which uses an adaptive gain control at the output of the classical echo canceller (CEC). A feedback loop permits approximate regulation of the front-edge CEC output power at a nominal level, independently of the echo and far-end signal levels. By this means, the precision required for adaptation is reduced to a minimum value. The analysis of adaptation, convergence, residual echo power, and computational complexity is given for the CGEC and compared to the similar quantities in a CEC; computer simulation results are presented. As an example, a 64 taps CGEC with only 16 bits instead of 20 can achieve secure binary data transmission (with bit error rate less than 10^{-6}) for a far-end signal-to-noise ratio of 16 dB and for an echo to far-end signal ratio of 20 dB, independently of the echo and far-end signal powers.

I. Introduction

THE conventional stochastic gradient algorithm has received a lot of applications in the area of optimum filtering. In particular, it is known to provide means for achieving reliable communications when the information signal is embedded in an additive noise with high power level, provided that a reference source correlated with the noise source [1] is available at the receiver.

In the field of data transmission, such a system is referred to as an "echo canceller." It is used especially for full-duplex data transmission through telephone lines where the mismatch of hybrid couplers between 2-wire and 4-wire circuits, and other channel impairments, are responsible for the presence of an undesirable echo whose power level may be as high as 20 dB above the information signal, hereafter simply denominated by "signal." In this context, the signal is the sequence s_k of far-end data, while the echo is a filtered copy σ_k' of the sequence a_k of locally transmitted data. Then the echo cancelling system is made of a transversal adaptive filter C_k which processes the sequence a_k and delivers the sequence σ_k of linearly estimated echo samples; this estimation is then subtracted from the observed input signal, resulting in the cancellation of the major part of the echo. Such classical echo cancellers (CEC) have been widely described in the literature [2]–[6].

Although the performances of these systems are quite satisfactory, it is well known that their hardware implementation requires an impressive number of bits for the representation of the adaptive filter coefficients. This is due to the wide dynamic ranges of the handled signals (echo or signal) and to the requirement of achieving a very low level of residual echo (e.g., 15–18 dB under the signal) for the purpose of subsequent correct far-end data recovery. For instance, a value of 20 bits is a minimum in echo cancellers for telephone channels, as a result of the constraints set upon the final error probability on far-end data and of the respective levels of maximum echo power and minimum information signal.

This problem of wide dynamics obviously requires methods of gain control. Since the signals are to be jointly processed by gain regulation and adaptive filtering for echo cancellation, the most interesting approach is to work jointly the algorithms for both treatments. Such an approach has already been used [7], with the adaptive gain control (AGC) located at several points in the block diagram of the whole receiver (far-end input signal, signal after echo cancellation, estimated far-end data). However, the resulting precision required in the CEC taps at each possible location has not been studied; moreover one important location for the AGC at the output of the adaptive filter and before echo cancellation has not been investigated.

The purpose of this paper is to introduce a new echo canceller with a reduced number of bits but identical performances as a CEC in terms of residual echo power. This is achieved by setting an AGC at the output of the CEC, with a feedback loop which ensures the necessary coupling between both. Thus, the output of the CEC can be regulated at a nominal predetermined level, independently of the actual signal and echo powers. By this means, the precision required for the adaptive coefficients is reduced to a minimum value which depends upon the desired signal-to-noise ratio at the canceller output. The gain in complexity is all the more significant as the ratio of echo-

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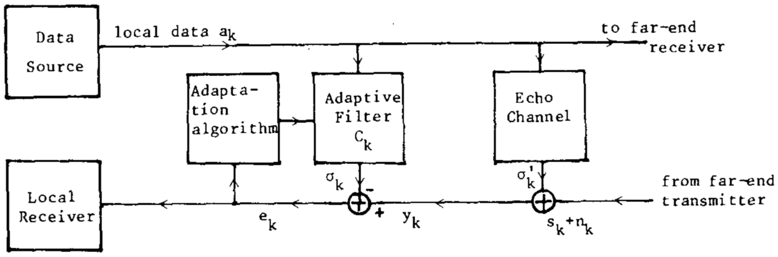

Fig. 1. Block diagram of a CEC.

to-signal powers is smaller. For instance, with real telephone channels, this is typical of distant echoes. The fact that near-end echoes will be treated by other means is not a real drawback since the different time locations of near-end and distant echoes will require the use of two distinct echo cancellers.

In the following, this system is denominated "controlled-gain echo canceller" (CGEC).

After the necessary recalls about the CEC in Section II, we present the new CGEC in Section III and discuss its convergence properties, residual echo level, and the gain in complexity. For both types of echo cancellers we report about the simulation results and their agreement with the theoretical discussion.

II. Classical Echo Cancellation

A. General Description of the CEC

A classical echo canceller intended for data transmission is depicted in Fig. 1.

Let T be the data symbol period, and let us call a_k, s_k, n_k, σ'_k, and y_k, respectively, the samples at time kT of local data, far-end data, channel noise, echo signal, and observed input signal. Denote respectively by C and C_k the sampled echo channel impulse response and the adaptive canceller tap vector at step k:

$$C_k = \left(C_1^k, C_2^k, \cdots, C_K^k\right)^T \tag{1}$$

where K is the length of these vectors. Defining the local data vector A_k through

$$A_k = \left(a_k, a_{k-1}, \cdots, a_{k-K+1}\right)^T, \tag{2}$$

one has for the actual and estimated echos, respectively,

$$\sigma'_k = C^T \cdot A_k; \qquad \sigma_k = C_k^T \cdot A_k, \tag{3}$$

while the error e_k used for adapting C_k is expressed through

$$e_k = y_k - \sigma_k = s_k + n_k + r_k, \tag{4}$$

where r_k is the residual echo

$$r_k = \sigma'_k - \sigma_k \tag{5}$$

after echo cancellation. The time variations of C_k follow the usual stochastic gradient algorithm [8]–[10].

$$C_{k+1} = C_k + \mu \cdot e_k \cdot A_k \tag{6}$$

which is suitable for baseband data transmissions where all the previously mentioned signals are real valued. In (6), the quantity μ, called step size, is a "small" positive constant. Let us call, respectively, S, A, N, P, and R the powers of far-end data s_k, local data a_k, channel noise n_k, echo signal σ'_k, and residual echo r_k. All these powers can be expressed in dB, when referenced to a given level, which is chosen here to be the local data level.

Moreover we assume that the data a_k are binary ($a_k = \pm 1$) and that the sequences $\{\cdots s_k \cdots\}$, $\{\cdots a_k \cdots\}$, and $\{\cdots n_k \cdots\}$ are white, zero-mean sequences, mutually independent; the latter assumption holds in most echo cancellation contexts.

The residual echo power which is defined by

$$R \triangleq \lim_{k\to\infty} E\left[\left|(C - C_k)^T \cdot A_k\right|^2\right] \tag{7}$$

is evaluated under the assumption that A_k and C_k are mutually independent vectors. Although this independence assumption is not realistic, it provides a result [11]–[12]

$$R = \frac{\mu K}{2 - \mu K} \cdot (S + N) \tag{8}$$

for the steady-state residual echo power, whose numerical value is known to be always in agreement [13] with practical applications. Then the SNR at the echo canceller output, defined as $\rho_s = S/(R+N)$, is such that

$$\rho_s = (2 - \mu K)\left(\mu K + \frac{2N}{S}\right)^{-1} < 2/(\mu K). \tag{9}$$

B. The CEC Implementation with Finite Precision

In practice, the tap coefficients (1) will have a binary representation in the basis

$$2^B, 2^{B-1}, \cdots, 2^0, \cdots, 2^{-b+1}, 2^{-b} \tag{9'}$$

where B and $-b$ are the respective orders of the most significant bit (MSB) and least significant bit (LSB). Including the sign bit, the total number of bits is thus

$$L = B + b + 2. \tag{10}$$

First, the integer B must be chosen in order to cover the whole dynamic range of the echo. Now the basis (9′) allows representation of values up to $2^{B+1} - 2^{-b} \simeq 2^{B+1}$. Assuming[1] that the echo excursion is limited to the range $(-2\sqrt{P}, 2\sqrt{P})$, one gets the condition

$$2^B \geqslant \sqrt{P}. \quad (11)$$

Secondly, the integer b must be chosen large enough to ensure that in each tap coefficient C_j^k, the algorithm increment $\mu e_k a_{k-j}$ will be taken into account in the digital implementation [15]–[17]. This means that

$$\mu \cdot |e_k a_{k-j}| \geqslant \frac{1}{2} 2^{-b}.$$

The quantity $|e_k a_k|$ can be approximated by its standard deviation. It derives from (4), (9) when the SNR at the echo canceller output satisfies the condition $\rho_s \gg 1$, and from the binary nature of a_k, that this inequality can be written

$$\mu \sqrt{S} \geqslant 2^{-b-1}. \quad (12)$$

Introducing the maximum echo power $P_{\max}$ and the minimum far-end data power $S_{\min}$, the number L of bits must satisfy

$$L \geqslant \log_2 \frac{1}{\mu} + \frac{1}{2} \log_2 \frac{P_{\max}}{S_{\min}} + 1. \quad (13)$$

In general, this number is very large. This point is demonstrated in the next section.

C. Computer Simulations

The computer simulations which have been run are relevant to a full-duplex data transmission at the bit rate of 1200 bits/s through a telephone channel (2-wire circuit). The baud rate is 1200 Hz and the echo channel, including the transmitting and receiving filters, corresponds to a raised cosine gain inside the bandwidth [−1200 Hz, 1200 Hz]. Notice that this range is transposed into the telephone range [600 Hz, 3000 Hz] by the intermediate modulation with the 1800 Hz carrier. The step-size value is held at $\mu = 10 \times 2^{-12.5}$ in a 1500 iterations preamble period, for the sake of quick initialization, during which the far-end transmitter is silent. Then an additional 1500 iterations period is allowed in the presence of double talking, holding μ at $2^{-12.5}$, which achieves the convergence. At the end of this period, the output SNR ρ_s is evaluated by averaging over 50 iterations.

The digitally implementable algorithm (6) is simulated on an IBM 360, all in integer arithmetic operations. In Fig. 2, ρ_s is plotted versus the number L of bits of the echo canceller taps, while the MSB is kept at the value $B = 0$. Two different cases are illustrated with identical ratios $P/S = 20$ dB, the SNR at the echo canceller input, denoted by ρ_e, being always $\rho_e = 20$ dB, but with different echo powers, namely $P = 0$ dB and $P = -20$ dB. It appears that a value very close to the ideal limiting value $\rho_s = \rho_e = 20$ dB is attained in the former case when $L = 17$ bits, while the latter case requires four additional bits ($L = 21$) to ensure the same value of ρ_s. This fact is in agreement with the theoretical result (13).

Another way of pointing out the problem is to consider the degradation of performances when the far-end data power S decreases in an echo canceller with a fixed number L of bits for its adaptive coefficients. This is illustrated through curve a of Fig. 3 for an echo canceller with $L = 16$ bits, the MSB being kept at $B = 0$. In Fig. 3, ρ_s is plotted versus the power S, the other parameters being kept constant at $P/S = 20$ dB, and $\rho_e = 16$ dB. It appears that ρ_s exhibits a relatively sharp maximum in the vicinity of $S = -18$ dB, while ρ_s closely approaches the input SNR

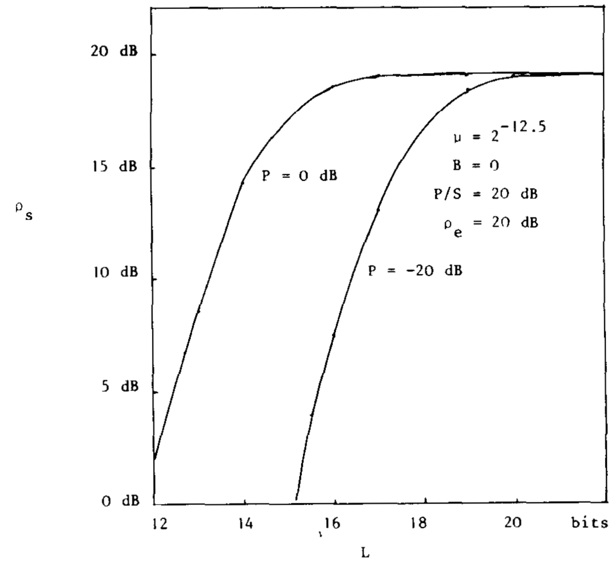

Fig. 2. SNR ρ_s versus number of bits for a CEC.

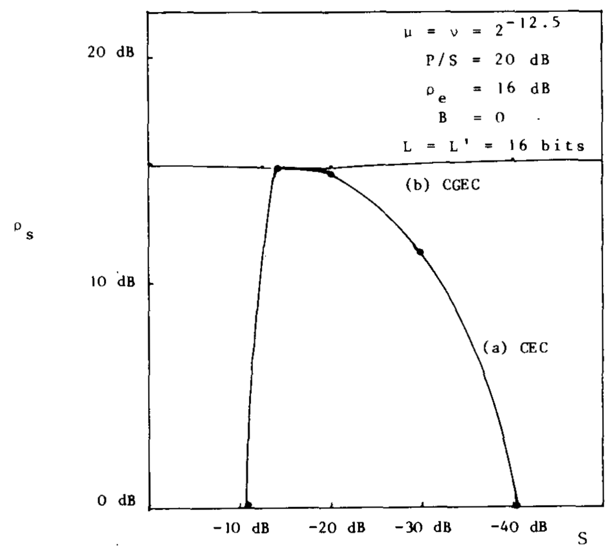

Fig. 3. Comparison of ρ_s between CEC and CGEC with fixed number of bits.

[1]This assumption is plausible because the echo corresponds to a finite weighted sum of the binary variables a_k; thus, it has a distribution more compact than a standard Gaussian variable.

TABLE I
ILLUSTRATION OF THE BINARY REPRESENTATION OF THE ADAPTIVE TAPS

$2^{B_{max}}, \ldots, 2^{B}, 2^{B-1}, \ldots, 2^{-p}, \ldots, 2^{-b}, \ldots, 2^{-b_{max}}$
$\sqrt{P_{max}}, \ldots, \sqrt{P}, \ldots, \ldots, \sqrt{S}, \ldots, 2^{-12}\sqrt{S}, \ldots, 2^{-12}\sqrt{S_{min}}$

useful bits (from $\sqrt{P}$ to $2^{-12}\sqrt{S}$)

$\rho_e = 16$ dB when the useful power S lies in this region; ρ_s breaks down when S decreases below -30 dB or increases above -12 dB; and ρ_s vanishes for very weak or strong far-end data powers.

The purpose of the new echo canceller to be presented now is to eliminate this drawback, by achieving the same performance for a wide range of signal powers, without an increase in the number of bits, e.g., with a standard number of 16 bits.

III. THE NEW ECHO CANCELLER WITH REDUCED COMPLEXITY

A. Introduction of the New Echo Canceller

Let us consider again the relations (11)–(13). Remembering that μ does not depend upon S and P, (12) expresses the fact that the LSB of the adaptive taps corresponds to a fixed downward shift below the signal level—for instance, 12 bits below $\sqrt{S}$. When (13) becomes an equality, the binary words have the exact useful length without unnecessary precision. Relations (11) and (12) are illustrated in Table I, with $\mu = 2^{-13}$, where p denotes the integer for which 2^{-p} is the closest to $\sqrt{S}$.

The useful bits are the 12 bits under the level $\sqrt{S}$ and the $[(1/2)\log_2(P/S)]$ bits above the level $\sqrt{S}$, in addition to the sign bit. Therefore, a total amount of

$$L' = \log_2 \frac{1}{\mu} + \frac{1}{2}\log_2\left(\frac{P}{S}\right)_{\max} + 1 < L \tag{14}$$

bits below the echo level $\sqrt{P}$ will, in any case, cover all the range of useful bits. However, the reduction from L to L' of the bit number implies the use of a floating scale, depending upon the echo level. This corresponds to identifying the ideal echo canceller C through a product of the form

$$C = F \cdot g \tag{15}$$

with

$$g = \sqrt{P}, \qquad \|F\| = 1, \tag{16}$$

as a result of the unitary property $E(a_k^2 = 1)$.

Therefore, the new echo canceller will consist of two parts.

- First a normalized filtering vector F_k, i.e.,

$$\|F_k\| = 1 \tag{17}$$

with the same dimension K as C; F_k changes according to an adaptive algorithm and converges to the ideal normalized vector F. It represents the direction of the echo response; F_k processes the binary data a_k, and its output

$$u_k = F_k^T A_k \tag{18}$$

has power regulated at the value 1:

$$E\left(u_k^2\right) = 1 \tag{19}$$

(or at a fixed level) through appropriate means which ensure that (17) or equivalently (19) is valid.

- Then a single multiplicative positive coefficient g_k which acts as an AGC and delivers the estimated echo according to

$$\sigma_k = g_k \cdot u_k. \tag{20}$$

Like F_k, g_k is adaptively incremented; it indicates the echo magnitude. Moreover the front-edge filter F_k is coupled with the subsequent g_k through a suitable loop. This explains the denomination of "controlled gain echo canceller" (CGEC) for the new system, whose block diagram is depicted in Fig. 4.

For practical purposes, it is of major importance that in the cascaded system (F_k, g_k), the high computational complexity device F_k acts directly on the binary signal a_k, like in a CEC. The adaptation algorithm of the new CGEC is therefore made of the joint system

$$(N) \begin{cases} F'_{k+1} = F_k + \dfrac{\mu}{g_k} e_k A_k & (21) \\ g'_{k+1} = g_k + \nu e_k u_k & (22) \\ F_{k+1} = \dfrac{F'_{k+1}}{\|F'_{k+1}\|}; \quad g_{k+1} = g'_{k+1}\|F'_{k+1}\|. & (23) \end{cases}$$

Notice that this algorithm ensures the normalization (17).

B. Convergence Properties of the CGEC

Let us now discuss the convergence properties of algorithm (N). This is a difficult point because the results available in the literature about the behavior of jointly adaptive algorithms in cascaded systems are very poor. In fact, the evaluation of the error at the output of such an adaptive system made of two jointly adaptive subsystems does not seem to have been performed in a general way and under well-defined assumptions. It is the purpose of the Appendix to compute this error in the specific case of the CGEC. With the same assumptions—see (A1), (A2)—as those generally used in data transmission, the residual echo power R' at the output of the CGEC

$$R' \triangleq \lim_{k \to \infty} E\left[\left(\sigma'_k - g_k F_k^T A_k\right)^2\right], \tag{24}$$

$$= \lim_{k \to \infty} E\left[\|C - F_k g_k\|^2\right] \tag{25}$$

is shown to be, for large K and small μK,

$$R' = \frac{1}{2}(\mu K + \nu)(S + N). \tag{26}$$

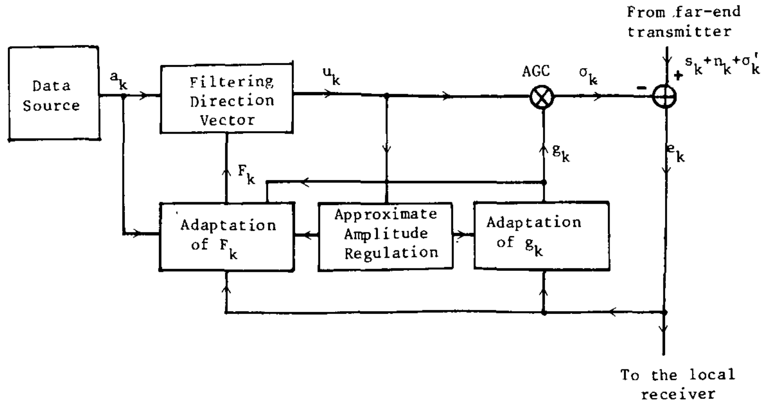

Fig. 4. The new CGEC.

This result shows that the residual echo can be made arbitrarily small by appropriate choice of μ and ν, according to a formula similar to (8), and that the error vector

$$V_k = F_k g_k - C \tag{27}$$

will converge to zero.

The constraint (23) that $\|F_k\| = 1$ thus implies the convergence of F_k to F and of g_k to $\sqrt{P}$. The result (26) is of importance. The comparison with (8), which is relevant to a CEC, shows that the effect of splitting the echo canceller into two adaptive parts the direction vector F_k and scalar gain g_k results, for the residual echo power, in an additive term that is proportional to the step size ν of the gain; the factor of increase is $(1+(\nu/\mu K))$. Thus, when K is very large and when ν is of the same order as μ, the residual echo is not appreciably increased.

Now it remains to prove that this new system provides a significant reduction in the echo canceller complexity.

C. The CGEC Computational Complexity

In the new system, the signal e_k after echo cancellation satisfies

$$e_k/\sqrt{P} = y_k/\sqrt{P} - F_k^T A_k, \tag{28}$$

once g_k has reached its asymptotic value $\sqrt{P}$. Thus, the adaptive filter F_k acts as an echo canceller for the echo $\sigma_k'/\sqrt{P}$ which has unit power and disturbs the signal $s_k/\sqrt{P}$ whose power is S/P. In order to fix the MSB ($2^{B'}$) and LSB ($2^{-b'}$) of the taps of F_k, we can apply the results of Section II-B, (11)–(13); whence

$$B' = 0; \qquad \mu\sqrt{\frac{S}{P}} \simeq 2^{-b'-1}, \tag{29}$$

and the total number L' of bits for the F_k taps is given by (14), according to our claim of Section III-A. Therefore, comparing (14) with (13), we conclude that, with respect to binary word length, the new echo canceller provides a reduction of

$$L - L' = \frac{1}{2}\left[\log_2\left(\frac{P_{\max}}{S_{\min}}\right) - \log_2\left(\frac{P}{S}\right)_{\max}\right] \tag{30}$$

bits over classical systems. The reduction of complexity follows from the fact that the range spanned by the ratio P/S is consistantly smaller than the value $(P_{\max}/S_{\min})$.

This is particularly true for distant echos. Indeed the power levels P and S depend mainly on the length of the respective echo and signal channels. For distant echos, the channel path includes a significant part of the transmission channel, and this distance is covered twice (once in each sense). Thus, the signal cannot be considerably more attenuated than the echo. For such echos, a practical situation could be, for instance,

$$P_{\max} = 0 \text{ dB}; \quad S_{\min} = -42 \text{ dB}; \quad (P/S)_{\max} = 20 \text{ dB} \tag{31}$$

corresponding to the case where the echo level reaches the maximum value of 0 dB; (31) is typical of a worst case since a power of 0 dB will be associated to a near-end echo rather than to a distant one. Then

$$L = 21 \text{ bits}; \qquad L' = 17 \text{ bits}. \tag{32}$$

In such a case the new CGEC offers a 4 bit reduction over a classical echo canceller, corresponding to a substantial gain of 20 percent.

D. Practical Implementation of the CGEC

In a practical realization, the CGEC will not implement exactly (21) and (23) because they both include a division. Actually, in the present state of the technological art, the digital hardware for a division is too costly, while a software computation means a lot of additional complexity. Fortunately, the regulation (19) of the output power of F_k, (or equivalently the regulation (17) of $\|F_k\|$) at the value 1, need not be accurate since it is only intended to bring at the level $B = 0$ the MSB of the estimated echo u_k. Hence, a coarse regulation within a range of one bit for u_k is sufficient. This will give a 2 bit regulation for the power, e.g.,

$$\frac{1}{2} \leqslant E(u_k^2) < 2. \tag{33}$$

Thus, without any degradation in the performances nor any increase in the bit number amount, the control (23) may be replaced by a simple shift of the binary representation of F_{k+1} and g_{k+1} according to (23′) below.

Similarly, in (21) the division by g_k needs not be accurate. Since it affects the step size of the algorithm only, g_k^{-1} can be replaced by a coarse estimate without influencing the limit of F_k. Hence, without any degradation, g_k will be replaced in (21) by its MSB 2^{p_k} and thus the division (e_k/g_k) is implemented by a simple shift of the binary word e_k. A practical realization of the new algorithm can therefore implement the set of equations

$$(N)' \begin{cases} u_k = F_k^T A_k & (18) \\ \sigma_k = g_k u_k & (20) \\ F'_{k+1} = F_k + \mu 2^{-p_k} e_k A_k \quad \text{if } 2^{p_k} \leqslant g_k < 2^{p_k+1} & (21') \\ g'_{k+1} = g_k + \nu e_k u_k & (22') \\ \left.\begin{array}{lll} F_{k+1} = F'_{k+1}; & g_{k+1} = g'_{k+1} & \text{if } \frac{1}{2} \leqslant E(u_k^2) < 2 \\ F_{k+1} = 2F'_{k+1}; & g_{k+1} = \frac{1}{2} g'_{k+1} & \text{if } E(u_k^2) < \frac{1}{2} \\ F_{k+1} = \frac{1}{2}F'_{k+1}; & g_{k+1} = 2g'_{k+1} & \text{if } E(u_k^2) \geqslant 2 \end{array}\right\}. & (23') \end{cases}$$

Let us now evaluate the additional operations required by the new systems $(N)'$ in comparison with a CEC as characterized by (3) and (6).

1) Apart from (23′) and the search for p_k, $(N)'$ has exactly the same number of operations as a CEC with $(K+1)$ adaptive taps (instead of K). The additional tap is the gain g_k; since u_k is regulated at the nominal power 1, the comparison of (22)′ and (6) shows that the binary representation of g_k requires the same number L of bits as the CEC tap, while each of the other K adaptive taps F_k in the CGEC offers a saving of $L - L'$ bits over the CEC.

2) The estimation of $E(u_k^2)$ required to implement the regulation (23′) implies at each step one multiplication and one addition. However, since we only need a coarse value of the power, these operations can be performed with a number of bits much lower than L or L'.

3) Finally, (21′) and (23′) imply a logical test of the MSB performed on g_k and $E(u_k^2)$, respectively. These tests need not be performed at each step, but periodically at the end of the averaging intervals used for the estimation of $E(u_k^2)$. In our computer simulations, this was done every 20 samples. In agreement with these logical tests, a shift of the binary representation of e_k [for (21′)] and of F'_{k+1} and g'_{k+1} [for (23′)] may be necessary from time to time. (Notice that in the steady state very few bit shifts will eventually be required.)

Because K is very large, the computational burden associated with the few arithmetical operations listed in 2) and 3) is very small and can be neglected in comparison with the proper echo cancellation itself described in 1). Therefore we conclude that the gain in complexity of the new echo canceller, with respect to a classical one is

$$G = L^{-1}\left[L\left(1 - \frac{1}{K}\right) - L'\right]. \quad (34)$$

In the practical situation (31) considered in this paper, with $K = 64$, $\mu = 2^{-12.5}$, and $\rho_e \geqslant 15$ dB, the gain G is 20 percent. Moreover, if one can accommodate with a value $\mu = 2^{-11.5}$ of the step size, due to a 1 dB increase in the input SNR ($\rho_e \geqslant 16$ dB) or a 1 dB decrease in the number K of taps, formulas (13) and (14) lead to

$$L = 20, \qquad L' = 16. \quad (35)$$

While the relative gain in complexity remains at the value 20 percent, the reduction of the absolute value of the binary word length from 20 to 16 is a very significant improvement since 16 bits is a standard size for hardware, while 20 is not.

E. Computer Simulations

The simulations of the CGEC which are described below are concerned with the same echo cancellation problems as described in Section II-C. In Fig. 5, the output SNR ρ_s is plotted versus the number L of bits of the direction vector taps, while the MSB is kept at the value $B' = 0$. The results must be compared with those of the CEC in Fig. 2, all the conditions being the same (identical μ, ρ_e, and P/S) for the two cases considered: $S = -20$ dB and $S = -40$ dB. Fig. 5 shows that in both cases, the number of bits required in the adaptive direction vector to achieve satisfactory performances is the same; namely with $L' = 17$ bits, the output SNR ρ_s reaches its maximum value very close to ρ_e.

The behavior of a CGEC with a fixed number of bits $L' = 16$ in the F_k taps is illustrated in Fig. 3(b), for different levels of the far-end data power S, the other parameters being kept constant ($\rho_e = 16$ dB, $P/S = 20$ dB). It appears that the plot of ρ_s versus S is a horizontal straight line. Hence, the new echo canceller achieves its goal and *provides satisfactory performances for constant P/S, with a reduced number of bits, independently of the far-end data power.*

Remark 1: A CGEC with a regulation of u_k within a range of 2 bits has also been simulated, by adopting the power range

$$\frac{1}{4} \leqslant E(u_k^2) < 4 \quad (33')$$

instead of (33), and the obvious corresponding changes in (23′). This change results in no significant degradation (less than 0.1 dB) in the output SNR, while it eliminates practi-

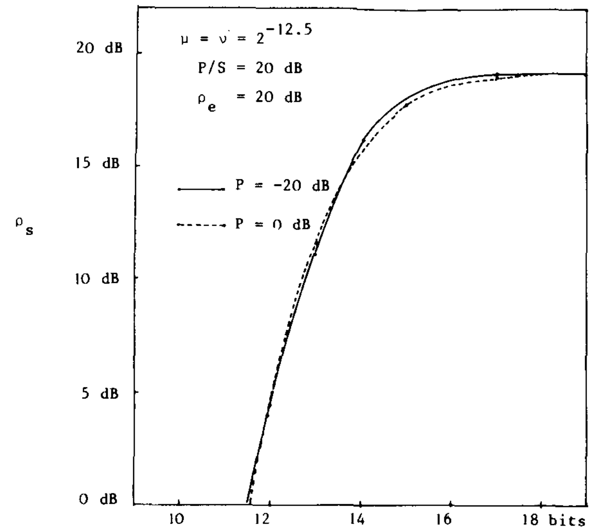

Fig. 5. Performance of CGEC versus the number of bits.

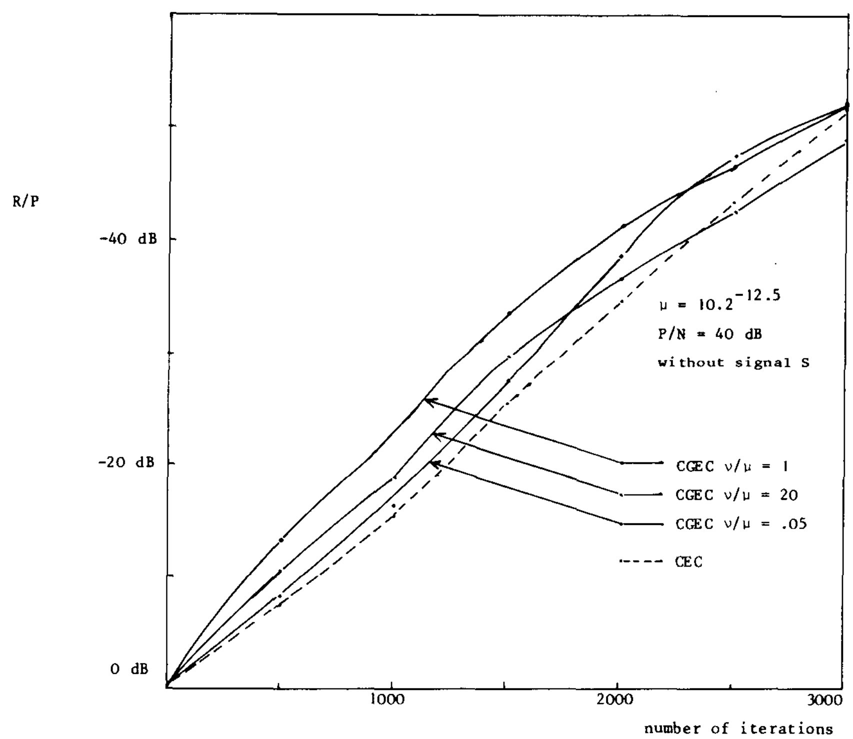

Fig. 6. Speed of convergence of the CGEC in terms of the ratio ν/μ.

cally all the binary shifts of the kind $F_{k+1} = 2F'_{k+1}$ during the steady state because the echo power fluctuations will always keep within the range (33′).

This emphasizes the statement, made previously, that a coarse regulation of $E(u_k^2)$ is sufficient.

Remark 2—Speed of Convergence: In addition to the gain in complexity which is expressed by the formulas (34), (35), the CGEC has another advantage over a CEC as a result from the additional degree of freedom which is provided to the system by the new convergence parameter ν.

This is illustrated through computer simulations in Figs. 6 and 7. Since we are only concerned with the transient behaviour of the echo canceller, we did not introduce the far-end data, which corresponds to the preamble learning period of the echo canceller. Then the noise in the adaptive system is only the additive channel noise N, whose level is taken at $N = -40$ dB, while the echo level is $P = 0$ dB.

In Fig. 6, the residual echo power R' is plotted versus time for different values of the ratio ν/μ of the step sizes.

An interesting period occurs between 1000 T and 2000 T, when the residual echo power is brought into the range

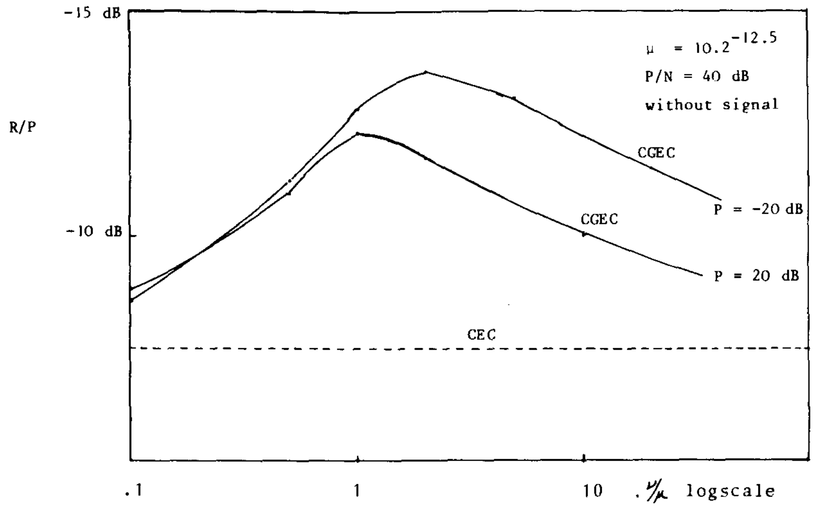

Fig. 7. R/P versus ν/μ after 500 iterations.

[-20 dB, -40 dB], that is below the far-end data level. Then it appears that the new echo canceller provides an 8 dB gain of attenuation over the classical one when the ratio ν/μ is set at the optimum, which occurs for the value 1. Such a gain is not a negligible additional advantage. Indeed, it corresponds to 400 iterations, i.e., roughly one-fourth of the preamble learning period, which can thus be shortened accordingly.

In Fig. 7 the echo attenuation is plotted after 500 iterations of the algorithm, versus the ratio ν/μ, for several values of the echo power ranging from -20 dB to 20 dB. The optimality of the value $\nu/\mu=1$ is confirmed by this figure, together with the 8 dB transient attenuation-improvement of the new system over the classical canceller.

IV. Conclusion

A new echo canceller structure has been discussed with the objective of reducing the binary length representation L of the adaptive taps. For that purpose the output power of the proper adaptive filtering vector (with large number of taps) is approximately regulated at a unit nominal level. Following the filtering itself, an adaptive gain control with feedback provides the estimated echo before cancellation. Without an appreciable increase in the amount of arithmetic, such a system reduces the value of L, if the ratio P/S of echo-to-signal powers is limited so that $P/S \ll P_{\max}/S_{\min}$. For instance, in real telephone channels, this is the case of distant echos. The corresponding gain increases with the ratio $P_{\max}/S_{\min}$. As an example, in the case of echo cancellation with 64 taps and minimum input SNR of 16 dB, satisfactory performance (e.g. a bit error rate of 10^{-6}) can be achieved with a reduced binary length of 16 instead of 20 bits, in the range of powers $P \leqslant 0$ dB; $S \geqslant -42$ dB; $P/S \leqslant 20$ dB. Moreover, the convergence speed is improved, and accordingly the learning period can be shortened by some 400 iterations.

Appendix
Evaluation of the Residual Echo Power of the CGEC

In this Appendix we prove formula (26) for the steady-state residual echo power R' of the new system. Our analysis uses the usual following assumptions.

Assumption A1: The sequences $\{\cdots, s_k, \cdots\}$, $\{\cdots, a_k, \cdots\}$, $\{\cdots, n_k, \cdots\}$ are zero-mean, white, and mutually independent.

Assumption A2: The data vector A_k is independent of the sequence $(F_1, g_1), \cdots, (F_k, g_k)$.

As already mentioned, Assumption A1 will usually be valid in practical situations. Now Assumption A2 will hold if, for instance, the successive vectors A_k are independent. Although this is far from realistic, it has already been emphasized by authors that the corresponding numerical results are correct (cf. Section II-A).

It derives from the equations (N) of the CGEC that

$$F_{k+1}g_{k+1} = F'_{k+1}g'_{k+1} = F_k g_k + \alpha_k + \beta_k + \gamma_k \quad \text{(A1)}$$

with

$$\alpha_k \triangleq \mu e_k A_k;$$
$$\beta_k \triangleq \nu e_k u_k F_k;$$
$$\gamma_k \triangleq \frac{\mu\nu}{g_k} u_k A_k e_k^2. \quad \text{(A2)}$$

Due to (17) and (19), α_k and β_k are of the same order of magnitude. Moreover, the ratio

$$\frac{\gamma_k}{\alpha_k} = \frac{\nu}{g_k} \cdot e_k u_k \simeq \frac{\mu}{g_k} e_k u_k \quad \text{(A3)}$$

will satisfy

$$\frac{\gamma_k}{\alpha_k} \ll 1, \qquad \forall k \quad \text{(A4)}$$

if

$$\frac{P}{S} \gg \frac{4}{K^2} \tag{A5}$$

in agreement with inequality (9). Now (A5) is always valid; otherwise there would be no need for an echo canceller, the echo level being far below the far-end data power.

For instance, when $K = 64$, (A5) is true whenever $P/S \geqslant -30$ dB. The opposite situation requires no echo cancellation. It derives from inequality (A4) that

$$W_{k+1} = W_k + \mu e_k \left[A_k + \frac{\nu}{\mu} u_k F_k \right] \tag{A6}$$

with the notation $W_k \triangleq F_k g_k$.

Defining the tap-coefficient error vector

$$V_k = F_k g_k - C \tag{A7}$$

it can be derived from (A6) that

$$V_{k+1} = U_k \cdot V_k + l_k (\nu u_k F_k + \mu A_k) \tag{A8}$$

where

$$U_k \triangleq I - \nu u_k F_k A_k^T - \mu A_k A_k^T,$$

I is the unit matrix, and $l_k \triangleq s_k + n_k$.

Before going into details, several notations are defined for ease of analysis.

$$\epsilon_k \triangleq E(V_k^T \cdot V_k), \qquad R' \triangleq \lim_{k \to \infty} E\left((A_k^T \cdot V_k)^2 \right), \tag{A9}$$

$$Q_k \triangleq E\left((V_k^T \cdot F)^2 \right), \tag{A10}$$

and the diagonal matrix D_k

$$D_k \triangleq \operatorname{diag}(\cdots, d_{ii}^k, \cdots) \triangleq \operatorname{diag}\left(\cdots, 1 - 2(f_i^k)^2, \cdots\right). \tag{A11}$$

Throughout this Appendix, vectors are designated by capital letters and their components by the corresponding lower case letters with subscript indices, e.g., vector V_k has components v_i^k, $i = 1, \cdots, K$. It appears easily that the residual echo power R' derives in a straightforward manner from the limiting value of ϵ_k, (A9), whose evaluation involves, in turn, the limiting value of Q_k, (A10).

1) Computing ϵ_k: The recursive equation (A8) is written again for convenience

$$V_{k+1} = U_k V_k + l_k \cdot (\nu u_k F_k + \mu A_k). \tag{A8$'$}$$

From the definition (A9) of ϵ_k

$$\epsilon_{k+1} = E(V_k^T U_k^T U_k V_k) + \mu^2 E(A_k^T A_k l_k^2) + \nu^2 E(F_k^T F_k u_k^2 l_k^2) + 2\mu\nu E(u_k^2 l_k^2), \tag{A12}$$

and the other terms are zero due to Assumptions A1 and A2. The first term in the RHS of (A12) is calculated as follows:

$$\begin{aligned} E(V_k^T U_k^T U_k V_k) - \epsilon_k &= -2\nu E(V_k^T u_k F_k A_k^T V_k) - 2\mu E(V_k^T A_k A_k^T V_k) \\ &\quad + \mu^2 E(V_k^T A_k A_k^T A_k A_k^T V_k) + \nu^2 E(V_k^T u_k^2 A_k F_k^T F_k A_k^T V_k) \\ &\quad + 2\mu\nu E(V_k^T u_k A_k F_k^T A_k A_k^T V_k). \end{aligned} \tag{A13}$$

Replacing in (A13) the term u_k by its value (18), we have to evaluate five terms. All these terms are evaluated in two steps. First, an inner average is performed with the statistics of the data vector A_k alone. Then, due to Assumption A2, the averages are made independently on the statistics of V_k, g_k, F_k; e.g.,

$$E(V_k^T A_k A_k^T V_k) = E(V_k^T E(A_k A_k^T) V_k)$$

and, due to Assumption A1,

$$E(V_k^T A_k A_k^T V_k) = E(V_k^T A V_k) = A \cdot \epsilon_k. \tag{A14}$$

The same technique is used in the whole Appendix. Thus, we get the recursive equation for ϵ_k.

$$\begin{aligned} \epsilon_{k+1} &= (1 - 2\mu A + \mu^2 K A^2)\epsilon_k - 2\nu[1 - A(2\mu + \nu)] A Q_k \\ &\quad + \nu(2\mu + \nu) A^2 E(V_k^T D_k V_k) \\ &\quad + (\mu^2 K + 2\mu\nu + \nu^2) A (S + N). \end{aligned} \tag{A15}$$

The recursive equation, already known [5], for the CEC is a particular case of (A15) when $\nu = 0$, as could be expected.

2) Computing Q_k: When the direction vector F_k has reached the steady state F, the quantity (A10) can be approximated through

$$Q_k = E\left((V_k^T F)^2 \right). \tag{A16}$$

Thus,

$$Q_{k+1} = E(F^T U_k V_k V_k^T U_k^T F) + E\left[l_k^2 F^T (\nu u_k F_k + \mu A_k)(\nu u_k F_k^T + \mu A_k^T) F \right].$$

We follow the same outlines as for evaluating ϵ_k. The recursive equation for Q_k is

$$Q_{k+1} = \left[1 - 2(\mu + \nu) A + 2(\mu + \nu)^2 A^2 \right] Q_k + (\mu + \nu)^2 A^2 E(V_k^T D_k V_k) + (\mu + \nu)^2 A (S + N). \tag{A17}$$

3) Computing $E(V_k^T D_k V_k)$: At the steady state, $E(V_k^T D_k V_k)$ can be approximated through

$$E(V_k^T D_k V_k) \simeq E(V_k^T D V_k) = \epsilon_k - 2E\left[\sum_l (v_l^k f_l)^2 \right], \tag{A18}$$

due to definition (A11). We remark that the quantity $E(V_k^T D_k V_k)$ appears in ϵ_{k+1} and Q_{k+1} (A15) and (A17) only in quadratic terms with respect to the step sizes μ, ν. Thus, it can be evaluated through the following approximation:

$$\left(v_l^k\right)^2 = \frac{1}{K}\|V_k\|^2 \tag{A19}$$

which expresses, at the first order, that the components of the error vector V_k all have the same distribution. Then from (A18), (A19)

$$E\left(V_k^T D_k V_k\right) = \left(1 - \frac{2}{K}\right)\epsilon_k. \tag{A20}$$

4) Computation of the Residual Echo Power R′: Replacing Q_k, Q_{k+1}, and ϵ_k, ϵ_{k+1} by their limiting values when $k \to \infty$, in (A15), (A17), and (A20), we get at the end

$$\begin{aligned} R' &= A \cdot \epsilon \\ &= \frac{\mu K + \nu + \nu(\mu + \nu)A}{2 - \left[\mu K + \nu\left(1 - \frac{2}{K}\right)\right]A - \nu(\mu + \nu)\left(1 - \frac{2}{K}\right)A^2} A \\ &\quad \cdot (S + N). \end{aligned} \tag{A21}$$

The approximate value of R' is given by

$$R' \cong \frac{1}{2}(\mu K + \nu)A(S + N)$$

which agrees with (26), when

$$K \gg 1; \qquad \mu K A \ll 1; \qquad \mu \approx \nu. \tag{A22}$$

These inequalities are usually satisfied in practice.

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